

Strong gravitational lensing in a noncommutative black-hole spacetime

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Abstract

Noncommutative geometry may be a starting point to a quantum gravity. We study the influence of the spacetime noncommutative parameter on the strong field gravitational lensing in the noncommutative Schwarzschild black-hole spacetime and obtain the angular position and magnification of the relativistic images. Supposing that the gravitational field of the supermassive central object of the galaxy described by this metric, we estimate the numerical values of the coefficients and observables for strong gravitational lensing. Comparing to the Reissner-Norström black hole, we find that the influences of the spacetime noncommutative parameter is similar to those of the charge, just these influences are much smaller. This may offer a way to distinguish a noncommutative black hole from a Reissner-Norström black hole, and may probe the spacetime noncommutative constant ϑ [1] by the astronomical instruments in the future.

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I. INTRODUCTION

The theoretical discovery of radiating black holes disclosed the first window on the mysteries of quantum gravity. Though after thirty years of intensive research, the full quantum gravity is still unknown. However there are two candidates for quantum gravity, which are the string theory and the loop quantum gravity. By the string/black hole correspondence principle [2], stringy effects cannot be neglected in the late stage of a black hole. In the string theory, coordinates of the target spacetime become *noncommuting* operators on a D -brane as [3]

$$[\hat{x}^\mu, \hat{x}^\nu] = i\vartheta^{\mu\nu}, \quad (1.1)$$

where $\vartheta^{\mu\nu}$ is a real, anti-symmetric and constant tensor which determines the fundamental cell discretization of spacetime much in the same way as the Planck constant \hbar discretizes the phase space, $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$. Motivated by string theory arguments, noncommutative spacetime has been reconsidered again and is believed to afford a starting point to quantum gravity.

Noncommutative spacetime is not a new conception, and coordinate noncommutativity also appears in another fields, such as in quantum Hall effect [4], cosmology [5], the model of a very slowly moving charged particle on a constant magnetic field [6], the Chern-Simon's theory [7], and so on. The idea of noncommutative spacetime dates back to Snyder [8] who used the noncommutative structure of spacetime to introduce a small length scale cut-off in field theory without breaking Lorentz invariance and Yang [9] who extended Snyder's work to quantize spacetime in 1947 before the renormalization theory. Noncommutative geometry [10] is a branch of mathematics that has many applications in physics, a good review of the noncommutative spacetime is in [11, 12].

The fundamental notion of the noncommutative geometry is that the picture of spacetime as a manifold of points breaks down at distance scales of the order of the Planck length: Spacetime events cannot be localized with an accuracy given by Planck length [12] as well as particles do in the quantum phase space. So that the points on the classical commutative manifold should then be replaced by states on a noncommutative algebra and the point-like object is replaced by a smeared object [13] to cure the singularity problems at the terminal stage of black hole evaporation [14].

The approach to noncommutative quantum field theory follows two paths: one is based on the Weyl-Wigner-

Moyal $*$ -product and the other on coordinate coherent state formalism [13]. In a recent paper, following the coherent state approach, it has been shown that Lorentz invariance and unitarity, which are controversial questions raised in the $*$ -product approach [15], can be achieved by assuming

$$\vartheta^{\mu\nu} = \vartheta \text{diag}(\epsilon_1, \dots, \epsilon_{D/2}), \quad (1.2)$$

where ϑ [1] is a constant which has the dimension of $length^2$, D is the dimension of spacetime [16] and, there isn't any UV/IR mixing. Inspired by these results, various black hole solutions of noncommutative spacetime have been found [17]; thermodynamic properties of the noncommutative black hole were studied in [18]; the evaporation of the noncommutative black hole was studied in [19]; quantized entropy was studied in [20], and so on.

It is interesting that the noncommutative spacetime coordinates introduce a new fundamental natural length scale $\sqrt{\vartheta}$. In this paper, we plan to study the influence of this constant on strong gravitational lensing.

The earlier studies of gravitational lensing have been developed in the weak field approximation [21]-[23]. It is enough for us to investigate the properties of gravitational lensing by ordinary stars and galaxies. However, when the lens is a black hole, a strong field treatment of gravitational lensing [24-29] is needed instead. Virbhadra and Ellis [26] find that near the line connecting the source and the lens, an observer would detect two infinite sets of faint relativistic images on each side of the black hole. These relativistic images could provide a profound verification of alternative theories of gravity. Thus, the study of the strong gravitational lensing becomes appealing recent years. On the basis of the Virbhadra-Ellis lens equation [27, 28], Bozza [30] extended the analytical method of lensing for a general class of static and spherically symmetric spacetimes and showed that the logarithmic divergence of the deflection angle at photon sphere is a common feature. Then Bhadra *et al* [31][32] have considered the Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole lensing. Eiroa *et al* [33] have studied the Reissner-Nordström black hole lensing. Konoplya [34] has studied the corrections to the deflection angle and time delay of black hole lensing immersed in a uniform magnetic field. Majumdar [35] has investigated the dilaton-de Sitter black hole lensing. Perlick [36] has obtained an exact lens equation and used it to study Barriola-Vilenkin monopole black hole lensing. S. Chen studied the K-S black hole lensing [37]. Bin-Nun [38] studied the strong gravitational lensing by Sgr A*, and so on.

The plan of our paper is organized as follows. In Sec. II we adopt to Bozza's method and obtain the deflection angles for light rays propagating in the noncommutative Schwarzschild black hole spacetime. In Sec. III we suppose that the gravitational field of the supermassive black hole at the centre of our galaxy can

be described by this metric and then obtain the numerical results for the observational gravitational lensing parameters defined in Sec. II. Then, we make a comparison between the properties of gravitational lensing in the noncommutative Schwarzschild and Reissner-Norström metrics. In Sec. IV, we present a summary.

II. DEFLECTION ANGLE IN THE NONCOMMUTATIVE SCHWARZSCHILD BLACK HOLE SPACETIME

The line element of the noncommutative Schwarzschild black hole reads [14]

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (2.1)$$

and

$$f(r) = 1 - \frac{4M}{r\sqrt{\pi}} \gamma(3/2, r^2/4\vartheta), \quad (2.2)$$

where $\gamma(3/2, r^2/4\vartheta)$ is the lower incomplete Gamma function:

$$\gamma(3/2, r^2/4\vartheta) \equiv \int_0^{r^2/4\vartheta} dt t^{1/2} e^{-t}, \quad (2.3)$$

ϑ is a spacetime noncommutative parameter [1]. The commutative Schwarzschild metric is obtained from (2.1) in the limit $r/\sqrt{\vartheta} \rightarrow \infty$. And Eq.(2.1) leads to the mass distribution $m(r) = 2M \gamma(3/2, r^2/4\vartheta) / \sqrt{\pi}$, where M is the total mass of the source. When $M > 1.9\sqrt{\vartheta}$, the event horizons are given by

$$r_{\pm} = \frac{4M}{\sqrt{\pi}} \gamma(3/2, r_{\pm}^2/4\vartheta), \quad (2.4)$$

which behaviors as that of Reissner-Norström black hole. The line element (2.1) describes the geometry of a noncommutative black hole and should give us useful insights about possible spacetime noncommutative effects on strong gravitational lensing.

As in [27, 28, 30], we set $2M = 1$ and rewrite the metric (2.1) as

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)(d\theta^2 + \sin^2 \theta d\phi^2), \quad (2.5)$$

with

$$A(r) = f(r), \quad B(r) = 1/f(r), \quad C(r) = r^2. \quad (2.6)$$

The deflection angle for the photon coming from infinite can be expressed as

$$\alpha(r_0) = I(r_0) - \pi, \quad (2.7)$$

where r_0 is the closest approach distance and $I(r_0)$ is [27, 28]

$$I(r_0) = 2 \int_{r_0}^{\infty} \frac{\sqrt{B(r)} dr}{\sqrt{C(r)} \sqrt{\frac{C(r)A(r_0)}{C(r_0)A(r)} - 1}}. \quad (2.8)$$

It is easy to obtain that as parameter r_0 decrease the deflection angle increase. At certain a point, the deflection angle will become 2π , it means that the light ray will make a complete loop around the compact object before reaching the observer. When r_0 is equal to the radius of the photon sphere, the deflection angle diverges and the photon is captured.

The photon sphere equation is given by [27, 28]

$$\frac{C'(r)}{C(r)} = \frac{A'(r)}{A(r)}, \quad (2.9)$$

which admits at least one positive solution and then the largest real root of Eq.(2.9) is defined as the radius of the photon sphere. To the noncommutative Schwarzschild black hole metric (2.1), the radius of the photon sphere can be given implicitly by

$$r_{ps} = \frac{3}{2} - \left[\frac{r_{ps}^3}{4\vartheta\sqrt{\pi\vartheta}} e^{-\frac{r_{ps}^2}{4\vartheta}} + \frac{3}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{r_{ps}^2}{4\vartheta}\right) \right], \quad (2.10)$$

which is an implicit function $f(r_{ps}, \vartheta) = 0$. It cannot be expressed as explicit function $r_{ps} = g(\vartheta)$, so we list some values of the photon sphere radius in the following table, and describe them in the Fig. 1. From the

TABLE I: Numerical values for the radius of the photon sphere in the noncommutative Schwarzschild black hole spacetime with different $\sqrt{\vartheta}$.

$\sqrt{\vartheta}$	0.260	0.254	0.248	0.242	0.236	0.230	0.224
r_{ps}	1.49151	1.49405	1.49593	1.49721	1.49824	1.49890	1.49934
$\sqrt{\vartheta}$	0.218	0.212	0.206	0.200	0.194	0.188	0.182
r_{ps}	1.49962	1.49979	1.49989	1.49995	1.49998	1.49999	1.50000

Tab. I, when $\sqrt{\vartheta} \rightarrow 0$, it can recovers that in the commutative Schwarzschild black hole spacetime which $r_{ps} = 1.5$. Fig. 1 shows that the relation between the photon sphere radius and the spacetime noncommutative parameter ϑ is very coincident to the function

$$r_{ps} = 1.5 - 7.8 \times 10^7 \sqrt{\vartheta}^{17}, \quad \sqrt{\vartheta} \in (0, \frac{1}{3.8}). \quad (2.11)$$

It is easy to see that this relation is quite different from that in the Reissner-Norström black hole spacetime $r_{ps} = (3 + \sqrt{9 - 32q^2})/4$, which implies that there exist some distinct effects of the noncommutative parameter ϑ on gravitational lensing in the strong field limit.

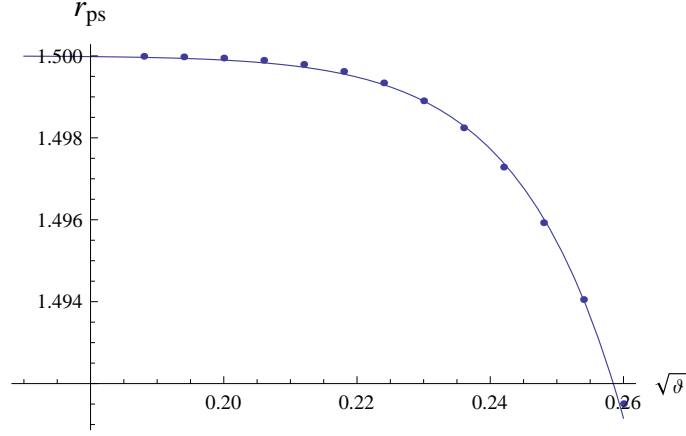


FIG. 1: The figure is for the radius of the photon sphere in the noncommutative Schwarzschild black hole spacetime with different $\sqrt{\vartheta}$. The dots are the exactly values described by Tab. I, the line is described by the expression $r_{ps} = 1.5 - 7.8 \times 10^7 \sqrt{\vartheta}^{17}$.

Following the method developed by Bozza [30, 37], we define a variable

$$z = 1 - \frac{r_0}{r}, \quad (2.12)$$

and obtain

$$I(r_0) = \int_0^1 R(z, r_0) f(z, r_0) dz, \quad (2.13)$$

where

$$R(z, r_0) = \frac{2r_0 \sqrt{A(r)B(r)C(r_0)}}{C(r)(1-z)^2} = 2, \quad (2.14)$$

$$f(z, r_0) = \frac{1}{\sqrt{A(r_0) - A(r)C(r_0)/C(r)}}. \quad (2.15)$$

The function $R(z, r_0)$ is regular for all values of z and r_0 . However, $f(z, r_0)$ diverges as z tends to zero. Thus, we split the integral (2.13) into two parts

$$\begin{aligned} I_D(r_0) &= \int_0^1 R(0, r_{ps}) f_0(z, r_0) dz, \\ I_R(r_0) &= \int_0^1 [R(z, r_0) f(z, r_0) - R(0, r_{ps}) f_0(z, r_0)] dz, \end{aligned} \quad (2.16)$$

where $I_D(r_0)$ and $I_R(r_0)$ denote the divergent and regular parts in the integral (2.13), respectively. To find the order of divergence of the integrand, we expand the argument of the square root in $f(z, r_0)$ to the second order in z and obtain the function $f_0(z, r_0)$:

$$f_0(z, r_0) = \frac{1}{\sqrt{p(r_0)z + q(r_0)z^2}}, \quad (2.17)$$

where

$$\begin{aligned} p(r_0) &= 2 - \frac{3}{r_0} + \frac{6}{\sqrt{\pi r_0}} \Gamma\left(\frac{3}{2}, \frac{r_0^2}{4\vartheta}\right) + \frac{r_0^2}{2\vartheta\sqrt{\pi\vartheta}} e^{-\frac{r_0^2}{4\vartheta}}, \\ q(r_0) &= \frac{3}{r_0} - 1 - \frac{6}{\sqrt{\pi r_0}} \Gamma\left(\frac{3}{2}, \frac{r_0^2}{4\vartheta}\right) - \frac{r_0^2}{4\vartheta\sqrt{\pi\vartheta}} e^{-\frac{r_0^2}{4\vartheta}} \left(2 + \frac{r_0^2}{2\vartheta}\right). \end{aligned} \quad (2.18)$$

When r_0 is equal to the radius of photon sphere r_{ps} , the coefficient $p(r_0)$ vanishes and the leading term of the divergence in $f_0(z, r_0)$ is z^{-1} , thus the integral (2.13) diverges logarithmically. Close to the divergence, Bozza [30] found that the deflection angle can be expanded in the form

$$\alpha(\theta) = -\bar{a} \log\left(\frac{\theta D_{OL}}{u_{ps}} - 1\right) + \bar{b} + O(u - u_{ps}), \quad (2.19)$$

where

$$\begin{aligned} \bar{a} &= \frac{R(0, r_{ps})}{2\sqrt{q(r_{ps})}} = \left[1 - \frac{r_{ps}^4}{8\vartheta^2\sqrt{\pi\vartheta}} e^{-\frac{r_{ps}^2}{4\vartheta}}\right]^{-\frac{1}{2}}, \\ \bar{b} &= -\pi + b_R + \bar{a} \log \frac{4q^2(r_{ps})[2A(r_{ps}) - r_{ps}^2 A''(r_{ps})]}{p'^2(r_{ps})u_{ps}r_{ps}\sqrt{A^3(r_{ps})}}, \\ b_R &= I_R(r_{ps}), \quad p'(r_{ps}) = \frac{dp}{dr_0}\bigg|_{r_0=r_{ps}}, \quad u_{ps} = \frac{r_{ps}}{\sqrt{A(r_{ps})}}. \end{aligned} \quad (2.20)$$

D_{OL} denotes the distance between the observer and the gravitational lens, \bar{a} and \bar{b} are so-called the strong field limit coefficients which depend on the metric functions evaluated at r_{ps} . In general, the coefficient b_R can not be calculated analytically and, in this case it cannot be evaluated numerically. Here we expand the integrand in (2.16) in powers of $\sqrt{\vartheta}$ as in [30]. Because the values of various low derivative of integrand of $I_R(r_{ps})$ at $\vartheta \rightarrow 0$ is zero, we can get

$$b_R = 2 \log[6(2 - \sqrt{3})] + O(\sqrt{\vartheta}). \quad (2.21)$$

Then we can obtain the \bar{a} , \bar{b} and u_{ps} , and describe them in Fig (2). Figures (2) tell us that with the increase of ϑ the coefficient \bar{a} increase, the \bar{b} slowly increases at first, then decrease quickly when it arrives at a peak, and the minimum impact parameter u_{ps} decreases, which is similar to that in the Reissner-Norström black hole metric. However, as shown in Fig. (2), in the noncommutative Schwarzschild black hole, \bar{a} increases more slowly, both of \bar{b} and u_{ps} decrease more slowly. In a word, comparing to the Reissner-Nordstrom black hole, the influences of the spacetime noncommutative parameter on the strong gravitational lensing is similar to those of the charge, merely they are much smaller. On the other side, in principle we can distinguish a noncommutative Schwarzschild black hole from the Reissner-Nordstrom black hole and, may be probe the value of the spacetime noncommutative constant by using strong field gravitational lensing.

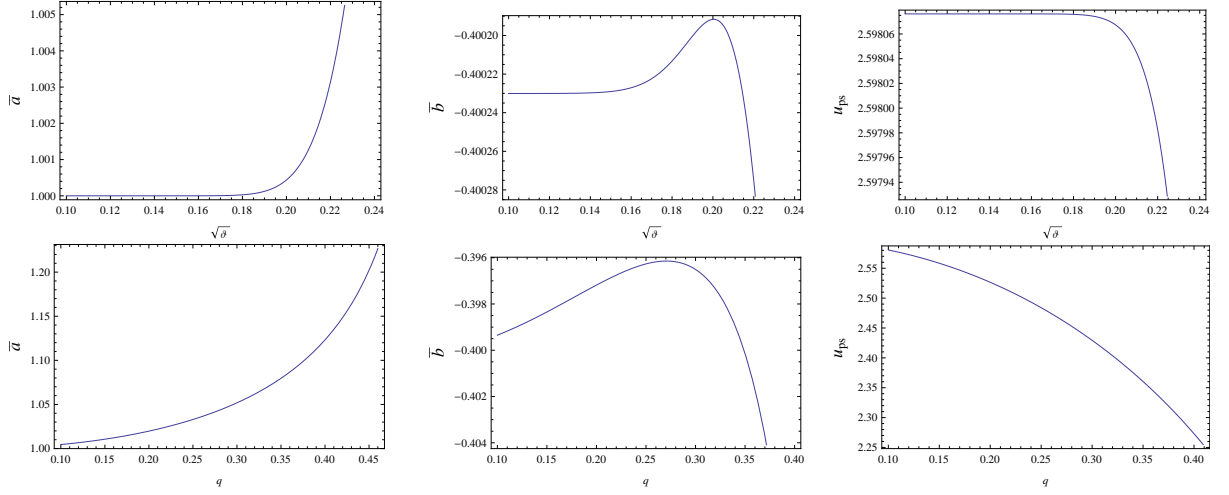


FIG. 2: Variation of the coefficients of the strong field limit \bar{a} , \bar{b} and the minimum impact parameter u_{ps} with the spacetime noncommutative parameter $\sqrt{\vartheta}$ in the noncommutative Schwarzschild black hole spacetime (in the upper row) and with q in the Reissner-Nordström black hole spacetime (in the lower row). The values of the coefficients of Reissner-Nordström lensing come from [30].

Figure (3) shows the deflection angle $\alpha(\theta)$ evaluated at $u = u_{ps} + 0.00326$. It indicates that the presence of ϑ increases the deflection angle $\alpha(\theta)$ for the light propagated in the noncommutative Schwarzschild black hole spacetime. Comparing with those in the commutative one, we could extract the information about the size of spacetime noncommutative parameter ϑ by using strong field gravitational lensing.

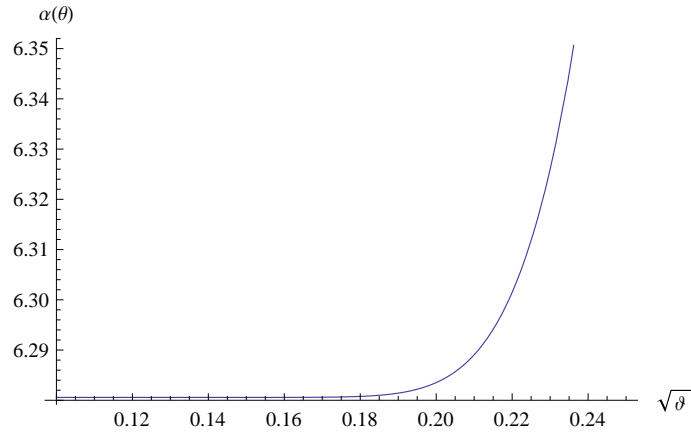


FIG. 3: Deflection angles in the noncommutative Schwarzschild black hole spacetime evaluated at $u = u_{ps} + 0.00326$ as functions of $\sqrt{\vartheta}$.

Considering the source, lens and observer are highly aligned, the lens equation in strong gravitational lensing can be written as [39]

$$\beta = \theta - \frac{D_{LS}}{D_{OS}} \Delta\alpha_n, \quad (2.22)$$

where D_{LS} is the distance between the lens and the source, $D_{OS} = D_{LS} + D_{OL}$, β is the angular separation

between the source and the lens, θ is the angular separation between the image and the lens, $\Delta\alpha_n = \alpha - 2n\pi$ is the offset of deflection angle and n is an integer. The position of the n -th relativistic image can be approximated as

$$\theta_n = \theta_n^0 + \frac{u_{ps}e_n(\beta - \theta_n^0)D_{OS}}{\bar{a}D_{LS}D_{OL}}, \quad (2.23)$$

where

$$e_n = e^{\frac{\bar{b}-2n\pi}{\bar{a}}}, \quad (2.24)$$

θ_n^0 are the image positions corresponding to $\alpha = 2n\pi$. The magnification of n -th relativistic image is given by

$$\mu_n = \frac{u_{ps}^2 e_n (1 + e_n) D_{OS}}{\bar{a} \beta D_{LS} D_{OL}^2}. \quad (2.25)$$

If θ_∞ represents the asymptotic position of a set of images in the limit $n \rightarrow \infty$, the minimum impact parameter u_{ps} can be simply obtained as

$$u_{ps} = D_{OL}\theta_\infty. \quad (2.26)$$

In the simplest situation, we consider only that the outermost image θ_1 is resolved as a single image and all the remaining ones are packed together at θ_∞ . Then the angular separation between the first image and other ones can be expressed as

$$s = \theta_1 - \theta_\infty, \quad (2.27)$$

and the ratio of the flux from the first image and those from the all other images is given by

$$\mathcal{R} = \frac{\mu_1}{\sum_{n=2}^{\infty} \mu_n}. \quad (2.28)$$

For highly aligned source, lens and observer geometry, these observable can be simplified as

$$\begin{aligned} s &= \theta_\infty e^{\frac{\bar{b}-2\pi}{\bar{a}}}, \\ \mathcal{R} &= e^{\frac{2\pi}{\bar{a}}}. \end{aligned} \quad (2.29)$$

The strong deflection limit coefficients \bar{a} , \bar{b} and the minimum impact parameter u_{ps} can be obtain through measuring s , \mathcal{R} and θ_∞ . Then, comparing their values with those predicted by the theoretical models, we can identify the nature of the black hole lens.

III. NUMERICAL ESTIMATION OF OBSERVATIONAL GRAVITATIONAL LENSING PARAMETERS

In this section, supposing that the gravitational field of the supermassive black hole at the galactic center of Milk Way can be described by the noncommutative Schwarzschild black hole metric, we estimate the numerical values for the coefficients and observables of the strong gravitational lensing, and then we study the effect of the spacetime noncommutative parameter ϑ on the gravitational lensing.

The mass of the central object of our Galaxy is estimated to be $2.8 \times 10^6 M_\odot$ and its distance is around 8.5 kpc. For different ϑ , the numerical value of the minimum impact parameter u_{ps} , the angular position of the asymptotic relativistic images θ_∞ , the angular separation s and the relative magnification of the outermost relativistic image with the other relativistic images r_m are listed in the table (II). It is easy to obtain that

TABLE II: Numerical estimation for main observables and the strong field limit coefficients for black hole at the center of our galaxy, which is supposed to be described by the noncommutative Schwarzschild black hole metric. R_s is Schwarzschild radius. $r_m = 2.5 \log \mathcal{R}$.

$\sqrt{\vartheta}$	$\theta_\infty (\mu\text{arcsecs})$	$s (\mu\text{arcsecs})$	$r_m (\text{magnitudes})$	u_{ps}/R_s	\bar{a}	\bar{b}
0	16.870	0.0211	6.8219	2.600	1.000	-0.4002
0.16	16.8699	0.02109	6.82188	2.59808	1.00000	-0.40023
0.18	16.8699	0.02110	6.82170	2.59808	1.00003	-0.40021
0.20	16.8698	0.02116	6.81890	2.59807	1.00044	-0.40019
0.22	16.8693	0.02154	6.80052	2.59798	1.00314	-0.40028
0.24	16.8662	0.02304	6.73143	2.59752	1.01344	-0.40058
0.26	16.8550	0.02759	6.54774	2.59579	1.04187	-0.40019

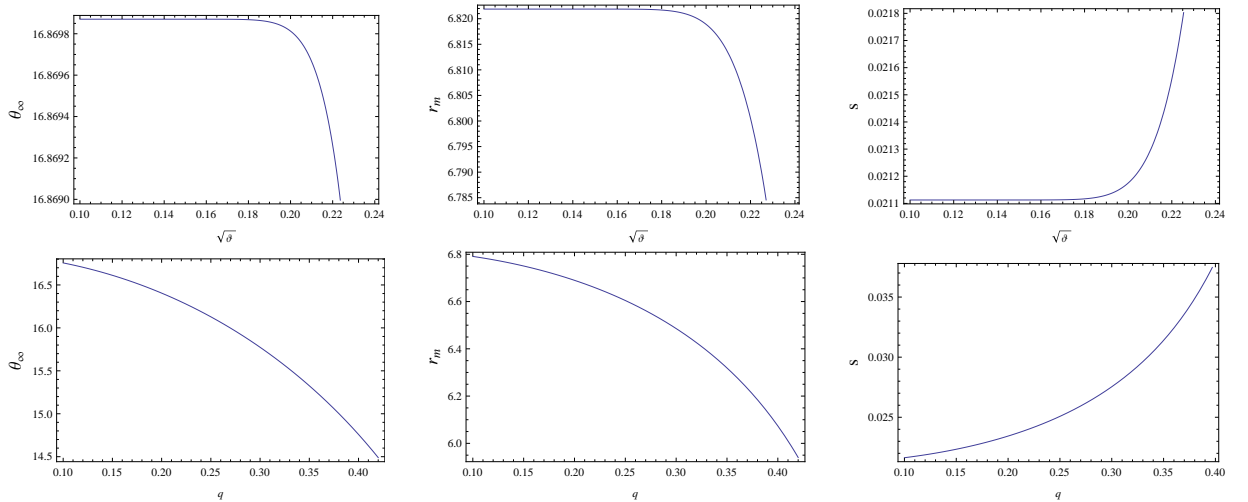


FIG. 4: Strong gravitational lensing by the Galactic center black hole. Variation of the values of the angular position θ_∞ , the relative magnitudes r_m and the angular separation s with parameter $\sqrt{\vartheta}$ in the noncommutative Schwarzschild black hole spacetime (in the upper row) and with q in the Reissner-Norström black hole (in the lower row).

our results reduce to those in the commutative Schwarzschild black hole spacetime as $\vartheta \rightarrow 0$. Moreover, from

the table (II), we also find that as the parameter ϑ increases, the minimum impact parameter u_{ps} , the angular position of the relativistic images θ_∞ and the relative magnitudes r_m decrease, but the angular separation s increases.

From Fig. (4), we find that in the noncommutative Schwarzschild black hole with the increase of parameter ϑ , the angular position θ_∞ and magnitudes r_m decreases more slowly, angular separation s increases more slowly than those in the Reissner-Norström black hole spacetime. This means that the bending angle is smaller and the relative magnification of the outermost relativistic image with the other relativistic images is bigger in the noncommutative Schwarzschild black hole spacetime. In order to identify the nature of these two compact objects lensing, it is necessary for us to measure angular separation s and the relative magnification r_m in the astronomical observations. Tables (II) tell us that the resolution of the extremely faint image is $\sim 0.03 \mu$ arc sec, which is too small. However, with the development of technology, the effects of the spacetime noncommutative constant ϑ on gravitational lensing may be detected in the future.

IV. SUMMARY

Noncommutative geometry may be a starting point to a quantum gravity. Spacetime noncommutative constant would be a new fundamental natural constant which can affect the classical gravitational effect such as gravitational lensing. Studying the strong gravitational lensing can help us to probe the spacetime noncommutative constant and the noncommutative gravity. In this paper we have investigated strong field lensing in the noncommutative Schwarzschild black hole spacetime to study the influence of the spacetime noncommutative parameter on the strong gravitational lensing. The model was applied to the supermassive black hole in the Galactic center. Our results show that with the increase of the parameter ϑ the minimum impact parameter u_{ps} , the angular position of the relativistic images θ_∞ and the relative magnitudes r_m decrease, and the angular separation s increases. Comparing to the Reissner-Norström black hole, we find that the angular position θ_∞ and magnitude r_m decrease more slowly, angular separation s increases more slowly. In a word, the influences of spacetime noncommutative parameter are similar to those of the charge, just they are much smaller. This may offer a way to distinguish a noncommutative Schwarzschild black hole from a Reissner-Norström black hole by the astronomical instruments in the future.

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- [1] The notation ϑ used here is a constant as well as Plank constant \hbar , but we still call it as a spacetime noncommutative parameter since it up to now is undetermined.
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